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TECHNICAL NOTE 2375

ON THE USE OF COUPLED MODAL FUNCTIONS IN FLUTTER ANALYSIS

By Donald S. Woolston and Harry L. Runyan

Langley Aeronautical Laboratory
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SUMMARY

An investigation of the flutter characteristics of a uniform, unswept, cantilever wing of high aspect ratio and under conditions of high mass coupling has been made by means of an analysis of the Rayleigh type based on coupled modal functions. Results are compared with experiment and also with the calculated results of NACA TN 1902 in which uncoupled modes were used. For the configuration studied, the use of coupled modes yielded, in general, no better agreement with experiment than did the use of uncoupled modes and, in some cases, the uncoupled-mode approach was better.

INTRODUCTION

In the study of flutter, many simplifications must be made in order to obtain practical solutions. One simplification common to most generally used methods of analysis is the use of a finite series of modal functions to represent wing motion during flutter. Of the many functions which could be used, either the coupled or the uncoupled modes of vibration of the system are usually chosen. A question, which naturally arises and one which has been a matter of practical interest to flutter analysts for some time, is that of the better choice of these two approaches. The coupled-mode approach has been found to be very time consuming compared with the uncoupled-mode approach. Various investigators (see, for example, reference 1), however, have expressed the thought that the use of coupled modes might give a higher degree of accuracy that would compensate for the greater amount of labor involved.

The present paper, which deals with the use of coupled modes, and references 2, 3, and 4 investigate this question. Reference 2 presents the experimental results of an extensive testing program made with an unswept, uniform, cantilever wing of fairly high aspect ratio. Flutter tests were made with a single concentrated weight mounted at various spanwise and chordwise positions on the wing.

In reference 3 a differential-equation analysis was applied to some of those cases of reference 2 where large mass coupling was involved. Good agreement between theory and experiment was obtained for all cases studied. The results indicated that the structural part of the problem was adequately taken into account by the differential-equation approach and that the theoretical two-dimensional aerodynamic coefficients were also adequate for the conditions investigated.

In reference 4 an analysis of the Rayleigh type, based on uncoupled-modal functions, was applied to a number of the experimental cases of reference 2. An effort was made to appraise the accuracy of this method of analysis and to determine the number of uncoupled modes needed to give a satisfactory result. No guide as to the number of modes which should be considered could be given and, in some cases, more than a practical number seemed necessary.

The present investigation is an extension of the work of these references and deals with the application of a Rayleigh type of analysis in which coupled modes are used. Analyses have been made for several spanwise stations of the case in reference 2 which is designated by weight 7 and leading-edge position a, as well as for the wing without a weight. Weight 7a was selected for analysis since, for this particular weight, the least satisfactory agreement with experiment was obtained by the uncoupled-mode approach (reference 4). The results of the present investigation in which coupled modes are used are compared with the results of the uncoupled-mode approach of reference 4. Since the results of reference 3 indicated that two-dimensional aerodynamic coefficients were adequate for these cases, this direct comparison of modal approximations seems possible.

A general outline of the procedure involved in conducting a coupled-mode analysis is given and includes the form of the flutter determinant. Application of the method to the specific cases is then discussed.

SYMBOLS

a — nondimensional distance of elastic axis from midchord measured in half-chords, positive for positions of elastic axis behind midchord

A_i — flutter-determinant element associated with kinetic and potential energies of mechanical system

b — wing half-chord

C_{ij} — flutter-determinant element associated with energies of air stream

f	flutter frequency, cycles per second
g	structural damping coefficient considered as variable in solution of flutter determinant
g_j	structural damping coefficient in jth coupled mode
h_i	bending component of ith coupled mode of vibration
I_α	mass moment of inertia per unit length referred to wing elastic axis
l	semispan of wing
L_h	aerodynamic wing lift coefficient due to bending oscillations of the wing (see reference 5)
L_α	aerodynamic wing lift coefficient due to torsional oscillations of the wing about its quarter chord (see reference 5)
M_h	aerodynamic moment coefficient about wing quarter-chord point due to bending oscillations of wing (see reference 5)
M_α	aerodynamic moment coefficient about the wing quarter-chord point due to torsional oscillations of wing about quarter-chord (see reference 5)
m	mass per unit length
S_α	static moment per unit length referred to wing elastic axis, positive for center of gravity behind elastic axis
v	flutter speed, feet per second
v_o	experimental flutter speed for wing without concentrated weight, 334 feet per second
$v/b\omega$	reduced flutter speed
x	spanwise coordinate measured from wing root
α_i	torsional component of ith coupled mode of vibration
ω	angular frequency at flutter, radians per second
ω_i	natural angular frequency of vibration in ith coupled mode

$\Omega_j = \frac{1}{\omega^2}\left(1 + ig_j\right)$, where i is imaginary quantity $\sqrt{-1}$

$\left.\begin{matrix}\phi_{h_1}, \phi_{h_2}, \\ \phi_{\alpha_1}, \phi_{\alpha_2}\end{matrix}\right\}$ uncoupled modal functions in first bending, second bending, first torsion, and second torsion, respectively

ρ mass density of air

Subscripts:

i, j designation of number of coupled modes; specific values 1, 2, and 3 used for a particular coupled mode

FLUTTER ANALYSIS WITH THE USE OF COUPLED MODES

The procedure for conducting a flutter analysis of the Rayleigh type for a given wing-weight configuration involves the selection of a set of modal functions to approximate the flutter mode, the formation of the flutter determinant, and the solution of this determinant for the flutter condition.

The modal functions usually employed are either the coupled or uncoupled modes of vibration of the system. The term "uncoupled mode", as employed in the present paper, refers to an imagined constrained mode in which, for pure bending, the chordwise distribution of mass is considered to act at the elastic axis of the wing with no torsional deformation occurring. For pure torsion, the elastic axis is considered restrained against bending. The term "coupled mode", as employed herein, refers to a combination of bending and torsional deflections appropriate to the natural harmonic vibrations of the freely oscillating undamped system.

For the purpose of the present analysis, coupled modes have been selected. These coupled modes may be determined in any of a number of ways (see, for example, appendix II of reference 6).

Once the coupled modes of the system are found, they are used together with the inertial characteristics of the system and the appropriate aerodynamic coefficients to form the flutter determinant. The complex flutter determinant, for the case of three coupled modes (see reference 6), may be given in the following form:

$$\begin{vmatrix} A_1\left(1 - \omega_1^{\,2}\Omega_1\right) + C_{11} & C_{12} & C_{13} \\ C_{21} & A_2\left(1 - \omega_2^{\,2}\Omega_2\right) + C_{22} & C_{23} \\ C_{31} & C_{32} & A_3\left(1 - \omega_3^{\,2}\Omega_3\right) + C_{33} \end{vmatrix} = 0$$

where the A_i's and C_{ij}'s are generalized constants which are computed from the inertial properties of the system, the coupled modes, and the aerodynamic coefficients and are given by:

$$A_i = \int_0^l m h_i^{\,2} dx + \int_0^l I_\alpha \alpha_i^{\,2} dx + 2\int_0^l S_\alpha h_i \alpha_i \, dx$$

$$\begin{aligned} C_{ij} = \pi\rho \Bigg\{ & \int_0^l b^2 L_h h_j h_i \, dx + \int_0^l b^3 \left[L_\alpha - L_h\left(\frac{1}{2} + a\right)\right]\alpha_j h_i \, dx + \\ & \int_0^l b^3 \left[M_h - \left(\frac{1}{2} + a\right)L_h\right] h_j \alpha_i \, dx + \\ & \int_0^l b^4 \left[M_\alpha - \left(\frac{1}{2} + a\right)L_\alpha - M_h\left(\frac{1}{2} + a\right) + L_h\left(\frac{1}{2} + a\right)^2\right]\alpha_j \alpha_i \, dx \Bigg\} \end{aligned}$$

The value ω_i is the angular frequency of the ith coupled mode of vibration. The parameter Ω is a characteristic value given, in terms of the flutter frequency ω and the concept of the structural damping coefficient g, by the relation

$$\Omega_j = \frac{1}{\omega^2}\left(1 + ig_j\right)$$

where i, in this expression, is the imaginary quantity $\sqrt{-1}$.

The functions h_i and α_i refer to the bending and torsional components, respectively, of the ith coupled mode.

The flutter condition is determined from the nontrivial solution of this determinant. This solution may be obtained by various methods (see, for example, reference 7).

APPLICATION AND DISCUSSION OF RESULTS

The method of analysis based on coupled modes discussed in the preceding section has been applied to the case in reference 2 where the wing weight was designated as 7a. This case represented a uniform, unswept, cantilever wing, 48 inches long, with a concentrated weight mounted at various spanwise positions and is the configuration for which an analysis based on uncoupled modes (reference 4) gave the least satisfactory agreement with experiment. The mass of the weight was of the same order as that of the wing. The position of the center of gravity of the weight was near the leading edge, well forward of the wing elastic axis. The calculations for flutter have been made for the wing without a weight and for the weight mounted at four different spanwise positions.

The coupled modes of vibration employed in the present investigation were obtained by a process of matrix iteration based on computed influence coefficients. The wing with distributed mass was considered as a system of concentrated masses located at 12 equally spaced stations along its span. Where a concentrated mass was included in the system, its effects were considered at the appropriate spanwise station. The procedure used was essentially that outlined in appendix II of reference 6. The actual iterative process was carried out on the Bell Telephone Laboratories X-66744 relay computer at the Langley Laboratory.

In the solution of the flutter determinant the structural damping coefficients were assumed equal so that

$$g_1 = g_2 = g_3 = g$$

where g is considered as a variable in the solution of the flutter determinant. The method of solution employed was that of reference 7. In this method the flutter determinant is put in the form of a set of simultaneous equations which are solved by a process of iteration. This iteration process yields values of g and ω from which the conventional plot of g against v can be obtained. For the cases considered herein, flutter conditions for the case $g = 0$ were obtained.

Analyses were made by use of the first three coupled modes of vibration of the system and by use of the three possible combinations of two coupled modes (that is, first and second, first and third, and second and third). Designation of the modes by numbers is on a frequency basis, with the first mode being that which occurs at the lowest frequency, and so forth. A comparison of experimental and calculated coupled and uncoupled frequencies is given in table I. Note that the first, second, and third coupled modes correspond primarily to uncoupled first bending, first torsion, and second bending, respectively, except in the case of the no-weight condition. The results of the analyses are compared with the calculated results of reference 4 and the experimental results of reference 2 in table II and figure 1. Where "no solution" is indicated in the table, either none exists or it is well beyond the range of practical significance. In agreement with reference 4 two uncoupled modes denotes uncoupled first bending and first torsion; three uncoupled modes denotes uncoupled first bending, first torsion, and second bending; and four uncoupled modes denotes uncoupled first bending, first torsion, second bending, and second torsion.

For the wing without a weight, good agreement between calculated and experimental results was obtained with both coupled and uncoupled modes. For this case the computed results were not particularly dependent on the number of modes considered. As shown in reference 2, the elastic axis of the wing was very near the center of gravity, so that for the wing without a weight very little mass coupling existed.

With the weight at the 11-inch spanwise station, the result obtained with two uncoupled modes $\left(v = 1.108\ v_o\right)$ is slightly better than that obtained with the three coupled modes $\left(v = 1.150\ v_o\right)$. Solutions were obtained for two of the three possible combinations of two coupled modes. When the first and second coupled modes (corresponding to uncoupled first bending and first torsion) were considered, a higher result $\left(v = 1.450\ v_o\right)$ was obtained. A solution was also obtained when the first and third coupled modes (corresponding to uncoupled first and second bending) were considered. The result for this analysis was very high $\left(v = 2.075\ v_o\right)$. All of these results were well above experiment $\left(v = 0.970\ v_o\right)$.

With the weight at the 17-inch spanwise station, the nearest approach to the experimental result $\left(v = 1.144\ v_o\right)$ was obtained with the analysis based on four uncoupled modes $\left(v = 1.491\ v_o\right)$. The use of the three coupled modes gave a higher answer $\left(v = 1.979\ v_o\right)$ which was, however, nearer the experimental value than the result obtained with three uncoupled modes $\left(v = 2.093\ v_o\right)$. No solution was obtained when only two uncoupled modes were considered. At this station a rather unexpected result was obtained in that an analysis based on two coupled modes (corresponding to the uncoupled first and second bending modes) gave a result $\left(v = 1.680\ v_o\right)$ which was in better agreement with experiment than that obtained with three coupled modes. The significance of this result is not clear at present.

For the weight at the 46-inch spanwise station, the analytical result based on three coupled modes $\left(v = 1.120\ v_o\right)$ and the result of the analysis based on the second and third coupled modes $\left(v = 1.078\ v_o\right)$ were both in close agreement with experiment $\left(v = 1.102\ v_o\right)$. For this case, the second and third coupled modes correspond to the uncoupled second bending and first torsion modes. Both of these results were considerably better than those of the uncoupled-mode analyses. When two uncoupled modes were considered, no solution was obtained. Consideration of three and four uncoupled modes both gave solutions, with that for three $\left(v = 1.260\ v_o\right)$ being slightly higher than that for four $\left(v = 1.228\ v_o\right)$.

With the weight at the 48-inch spanwise station (tip) the analysis based on three coupled modes gave a result $\left(v = 1.072\ v_o\right)$ in close agreement with the results of the analyses based on three and four uncoupled modes $\left(v = 1.060\ v_o \text{ and } v = 1.063\ v_o\text{, respectively}\right)$, all of these being above the experimental result $\left(v = 0.958\ v_o\right)$. When the second and third coupled modes (corresponding to uncoupled second bending and first torsion) were considered, a result $\left(v = 1.018\ v_o\right)$ in slightly better agreement with experiment was obtained. For this station, too, no solution was obtained from the analysis based on uncoupled first bending and first torsion.

With very few exceptions analysis of these cases by either the coupled- or the uncoupled-mode approach gave results which were high in comparison with experiment.

Figure 1 and table II indicate that increasing the number of modes in the coupled-mode analyses of these cases did not usually cause the result to converge toward the experimental results. In the uncoupled-mode approach of reference 4 the addition of a mode generally caused the result to converge toward experiment.

When economy of computing time is considered, the uncoupled-mode approach is by far the better method. As an example, if only the determination of the modes is considered, the computation of three coupled modes by the automatic computing methods employed required approximately 23 hours and would correspond to a minimum of 70 hours of manual computing. In contrast to this, the time required to compute manually three uncoupled modes would be approximately 5 hours. The amount of time and labor required in the formation of the flutter determinant for the coupled-mode approach is also greater than that for the uncoupled-mode approach.

For the cases studied, the use of coupled modes yielded, in general, no better agreement with experiment than did the use of uncoupled modes and, in some cases, the uncoupled-mode approach was better. This result is in conflict with the frequently expressed thought that coupled modes, being more realistic insofar as the ground-vibration modes are concerned, should give a better approximation to the flutter mode and, therefore, a better result in flutter calculations. It should be recognized that the calculations performed were conducted for a uniform, unswept, cantilever wing and that the conclusions may not necessarily be applicable to a sweptback wing. The observation that uncoupled modes give as good results as and, in some cases, better results than coupled modes is quite interesting. The reason for this result, however, is not known but it may be caused by the allowance of more freedom for the uncoupled-mode analysis to combine the bending and torsion modes. In the coupled-mode analysis, a mode has both a bending and a torsional component and these components are in phase and have fixed relative amplitudes. This phase and amplitude relation may cause some restriction in the combining of the modes and may be detrimental.

CONCLUSIONS

An investigation of the flutter characteristics of a uniform, unswept, cantilever wing of high aspect ratio and under conditions of high mass coupling has been made by means of an analysis of the Rayleigh type based on coupled modal functions. From the comparison presented herein of the results of the coupled-mode analysis with those of the uncoupled-mode analysis of NACA TN 1902 and with experiment, the following conclusions can be drawn (which may not necessarily be applicable to a sweptback wing):

1. In most of the cases considered, which were selected to make a rather severe test of the use of modal functions, the use of either the coupled- or the uncoupled-mode approach gave results which were high in comparison with experiment.

2. Increasing the number of modes in the coupled-mode analyses of these cases did not usually cause the result to converge toward the experimental results. Such convergence with added uncoupled modes was indicated in NACA TN 1902.

3. In comparison with the coupled-mode approach, the uncoupled-mode approach was by far the better method when economy of computing time is considered.

4. For the cases treated herein, the use of coupled modes yielded, in general, no better agreement with experiment than did the use of uncoupled modes and, in some cases, the uncoupled-mode approach was better.

Langley Aeronautical Laboratory
National Advisory Committee for Aeronautics
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REFERENCES

1. Scanlan, R. H.: Discussion of "The Flutter of a Uniform Wing with Tip Weights." Jour. Appl. Mech., vol. 15, no. 4, Dec. 1948, pp. 387-388.

2. Runyan, Harry L., and Sewall, John L.: Experimental Investigation of the Effects of Concentrated Weights on Flutter Characteristics of a Straight Cantilever Wing. NACA TN 1594, 1948.

3. Runyan, Harry L., and Watkins, Charles E.: Flutter of a Uniform Wing with an Arbitrarily Placed Mass According to a Differential-Equation Analysis and a Comparison with Experiment. NACA Rep. 966, 1950.

4. Woolston, Donald S., and Runyan, Harry L.: Appraisal of Method of Flutter Analysis Based on Chosen Modes by Comparison with Experiment for Cases of Large Mass Coupling. NACA TN 1902, 1949.

5. Smilg, Benjamin, and Wasserman, Lee S.: Application of Three-Dimensional Flutter Theory to Aircraft Structures. ACTR No. 4798, Materiel Div., Army Air Corps, July 9, 1942.

6. Scanlan, R. H., and Rosenbaum, Robert: Outline of an Acceptable Method of Vibration and Flutter Analysis for a Conventional Airplane. Airframe and Equipment Eng. Rep. No. 43, C.A.A., U. S. Dept. Commerce, Oct. 29, 1948.

7. Wasserman, Lee S., Mykytow, Walter J., and Spielberg, Irvin: Tab Flutter Theory and Applications. AAF TR No. 5153, Air Technical Service Command, Army Air Forces, Sept. 1, 1944.

TABLE I.- EXPERIMENTAL AND CALCULATED FREQUENCIES

Weight	Span position (in. from root)	Frequency (cps)								
		First mode (a)			Second mode (b)			Third mode (c)		
		Experimental	Uncoupled	Coupled	Experimental	Uncoupled	Coupled	Experimental	Uncoupled	Coupled
None	0	6.25	6.55	6.63	35.8	41.7	41.2	44.6	48.5	48.6
7a	11	6.20	6.53	6.34	22.8	28.1	25.4	34.1	----	39.0
7a	17	5.90	6.26	6.21	19.7	23.0	20.7	(d)	26.9	35.5
7a	46	3.16	3.16	3.17	17.6	14.7	18.5	31.8	33.4	33.6
7a	48	2.70	3.09	3.06	18.0	14.5	18.2	29.8	31.5	31.9

[a]Primary component of deflection similar to first cantilever bending mode.

[b]Primary component of deflection similar to second cantilever bending mode in no-weight condition but similar to first cantilever torsional mode for all other weight conditions.

[c]Primary component of deflection similar to first cantilever torsional mode in no-weight condition but similar to second cantilever bending mode for all other weight conditions.

[d]Not clear.

TABLE II.- EXPERIMENTAL AND CALCULATED RESULTS

(a) Uncoupled modes.

Weight	Span position (in. from root)	Experimental results (reference 2)			Calculated results for uncoupled modes (reference 4)								
					Two modes $\phi_{h_1}, \phi_{\alpha_1}$			Three modes $\phi_{h_1}, \phi_{h_2}, \phi_{\alpha_1}$			Four modes $\phi_{h_1}, \phi_{h_2}, \phi_{\alpha_1}, \phi_{\alpha_2}$		
		f (cps)	$v/b\omega$	v/v_o (a)	f (cps)	$v/b\omega$	v/v_o (a)	f (cps)	$v/b\omega$	v/v_o (a)	f (cps)	$v/b\omega$	v/v_o (a)
None	0	22.1	7.22	1.000	25.2	6.10	0.961	23.9	6.8	1.018	----	----	----
7a	11	17.4	8.88	.970	21.3	8.28	1.108	----	----	----	----	----	----
7a	17	[b]16.3 26.8	[b]11.04 7.02	1.144	(c)	(c)	(c)	14.21	23.52	2.093	32.8	7.27	1.491
7a	46	21.8	8.09	1.102	(c)	(c)	(c)	18.00	7.64	1.260	18.2	7.37	1.228
7a	48	21.4	7.14	.958	(c)	(c)	(c)	25.50	6.65	1.060	24.8	6.06	1.063

(b) Coupled modes.

Weight	Span position (in. from root)	Experimental results (reference 2)			Calculated results for coupled modes											
					Two modes									Three modes		
					1st and 2nd			1st and 3rd			2nd and 3rd					
		f (cps)	$v/b\omega$	v/v_o (a)	f (cps)	$v/b\omega$	v/v_o (a)	f (cps)	$v/b\omega$	v/v_o (a)	f (cps)	$v/b\omega$	v/v_o (a)	f (cps)	$v/b\omega$	v/v_o (a)
None	0	22.1	7.22	1.000	(c)	(c)	(b)	24.2	6.67	1.009	(c)	(c)	(c)	24.2	6.76	1.024
7a	11	17.4	8.88	.970	14.5	15.93	1.450	35.6	9.28	2.075	(c)	(c)	(c)	18.9	9.68	1.150
7a	17	[b]16.3 26.8	[b]11.04 7.02	1.144	(c)	(c)	(c)	30.0	8.93	1.680	(c)	(c)	(c)	30.6	10.30	1.979
7a	46	21.8	8.09	1.102	(c)	(c)	(c)	(c)	(c)	(c)	24.3	7.06	1.078	25.0	7.14	1.120
7a	48	21.4	7.14	.958	(c)	(c)	(c)	(c)	(c)	(c)	23.6	6.88	1.018	24.7	6.92	1.072

[a] v_o = 334 fps.

[b] In reference 2 with the weight at 17 inches from the root section the wing appeared to diverge. However, the oscillograph records for this case showed two possible flutter points, one corresponding to a frequency of 16.3 cps (rather than the value of 16.0 cps recorded in reference 2) and another corresponding to a frequency of 26.8 cps. Only the first of these values is noted in reference 2.

[c] No solution.

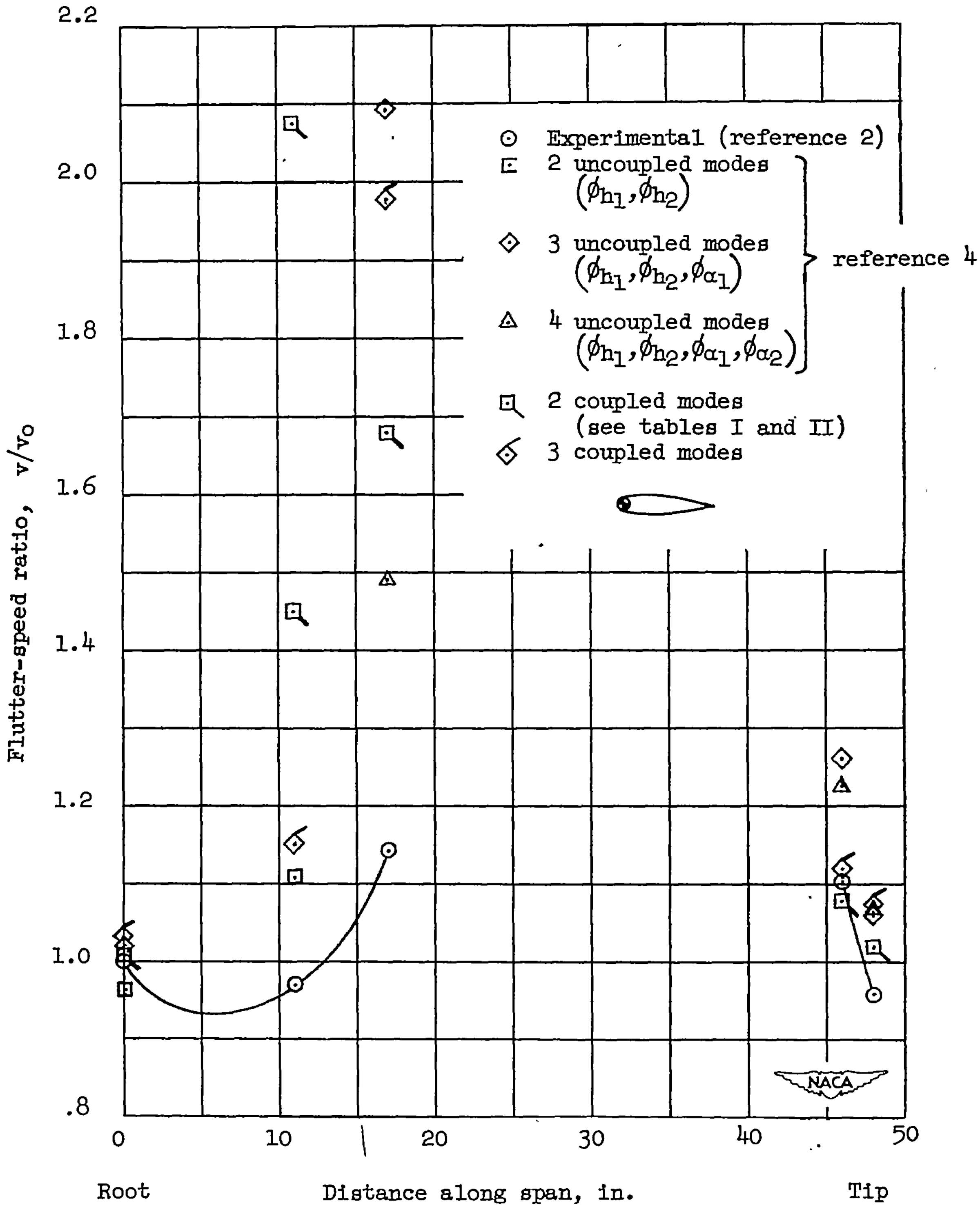

Figure 1.- Comparison of calculated and experimental flutter speeds for a particular wing-weight system (weight 7a of reference 2).

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